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A stringy cloak for a classical singularity

Atish Dabholkar

Department of Theoretical Physics, Tata Institute of Fundamental Research
Homi Bhabha Road, Mumbai 400005, India
Email: atish@tifr.res.in

Renata Kallosh[a] and Alexander Maloney[ab]

[a] *Institute for Theoretical Physics, Department of Physics, Stanford University*
Stanford, CA 94305, U.S.A., and
[b] *Theory Group, Stanford Linear Accelerator Center*
2575 Sand Hill Rd, Menlo Park, CA 94025, U.S.A.
Email: kallosh@stanford.edu, maloney@slac.stanford.edu

Abstract: We consider a class of 4D supersymmetric black hole solutions, arising from string theory compactifications, which classically have vanishing horizon area and singular space-time geometry. String theory motivates the inclusion of higher derivative terms, which convert these singular classical solutions into regular black holes with finite horizon area. In particular, the supersymmetric attractor equations imply that the central charge, which determines the radius of the $AdS_2 \times S^2$ near horizon geometry, acquires a non-vanishing value due to quantum effects. In this case quantum corrections to the Bekenstein-Hawking relation between entropy and area are large. This is the first explicit example where stringy quantum gravity effects replace a classical null singularity by a black hole with finite horizon area.

Keywords: Black Holes in String Theory, Black Holes, Supergravity Models.

Contents

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1. Introduction

In any quantum theory of gravity, the low-energy effective action is expected to include higher derivative corrections to the Einstein-Hilbert action. In string theory, such corrections can be computed in principle and in many cases may be found explicitly. We will argue that these corrections play an important role in the physics of certain black holes. Many extremally charged black holes and branes have null singularities that are not separated from asymptotic observers by a regular horizon. For a broad class of examples, however, the higher derivative terms completely alter the causal structure of these objects, transforming these apparently singular solutions into regular black holes with finite horizon area. After this 'stringy cloaking', the near horizon geometry becomes $AdS_2 \times S^2$.

We will work in four dimensional, ungauged $N = 2$ supergravity [1]. In this case the BPS black hole solutions exhibit fixed-point attractor behavior near the horizon, as discovered by Ferrara, Kallosh and Strominger [2, 3]. Cardoso, de Wit and Mohaupt described how higher order derivative terms, representing quantum effects, may be added to the action, and found the generalized attractor equations in the presence of these additional terms [4]. With Kappeli, they discussed the general class of stationary BPS solutions, which includes ones with classicaly vanishing horizon [5]. We will follow the work of [4, 5] closely in what follows. Wald has proposed a modification to the usual Bekenstein-Hawking area law [6] in the presence of higher curvature terms [7], which was applied to these black holes by [4]. This work focused primarily on black holes that already have a regular horizon and large area in the leading supergravity approximation, where the entropy receives small but calculable corrections from higher derivative terms, although solutions with classicaly vanishing area were also considered. Remarkably, the leading order correction agrees with the counting of black hole microstates performed by Maldacena, Strominger and Witten, and by Vafa [8]. Recently, Ooguri, Strominger and Vafa discussed an interesting relation between these black holes and topological string theory [9].

The effects of the higher derivative terms are most dramatic for black holes that classically have zero area and hence zero entropy. It was noticed recently by Dabholkar [10] that for certain two-charge black holes of this type occurring in Type-IIA theory compactified on $K_3 \times T^2$, the correction to the entropy coming from higher derivative prepotential-like terms reproduces precisely the logarithm of the number of BPS microstates [11, 12].

Motivated by these results, we examine the space-time geometry of a general class of black holes in N=2 supergravity with higher derivative corrections. In particular, we consider solutions of Type-IIA string theory compactified on a general Calabi-Yau 3-fold, of which the two-charge black hole mentioned above is a special case. We will focus on solutions which classically have null singularities and zero horizon area. We will argue that in these cases the higher curvature terms cause the black holes to develop a regular horizon with non-zero area.[1] In Einstein frame, the radius of the horizon is large for large charges. In addition to making the area non-zero, the higher curvature terms significantly modify the Bekenstein-Hawking relation between entropy and area. The net entropy S is then related to the corrected area A by the relation $S = A/2$, which is distinctly different from the standard relation $S = A/4$.

The paper is organized as follows. In section 2, we review the basic formalism of the generalized attractor mechanism and describe black hole solutions for Calabi-Yau compactifications. More details can be found in [14]. In section 3 we show that for many choices of electric and magnetic charges the classical geometry has a null singularity which is replaced by a smooth horizon once the corrections are taken into account. We discuss some implications when the Calabi-Yau 3-fold is $K_3 \times T^2$ in section 4, and summarize our findings in section 5.

2. Review

2.1 Higher curvature corrections to $N = 2$, $D = 4$ supergravity

Our starting point is the $N = 2$ superconformal theory of gravity in four dimensions described by [1], which is subsequently gauge fixed to give $N = 2$ Poincaré supergravity. The field content is a Weyl multiplet, which contains the graviton as well as other physical and auxiliary fields, along with $n + 1$ vector multiplets. One of these vector fields is the graviphoton, which is not contained in the Weyl multiplet. One can define a superfield $\mathbf{W}^{ij}_{ab}$, where $i, j = 1, 2$ are spinor indices and a, b are local Lorentz indices, which contains the field strengths of the Weyl multiplet as well as an anti-selfdual auxiliary field $T^{ab\,ij}$. The lowest component of the associated chiral multiplet $\mathbf{W}^2 = (\mathbf{W}^{ab\,ij}\varepsilon_{ij})^2$ is $\hat{A} = (T^{ab\,ij}\varepsilon_{ij})^2$. The couplings between the vector multiplets and the Weyl multiplet are summarized by a holomorphic function

$$F(X^I, \hat{A})\,, \qquad I = 0, 1, \ldots, n \tag{2.1}$$

[1]This result is similar in spirit to the enhancon mechanism of Johnson, Peet and Polchinski [13], although the mechanism is quite different.

which is homogeneous of degree two. When F is independent of $\hat{A}$, this is the usual holomorphic prepotential, which depends on the scalar components X^I of the $n+1$ vector multiplets.

The supergravity field equations are covariant under symplectic $\mathrm{SP}(2n+2;\mathbb{Z})$ transformations, which are a generalization of electric-magnetic duality. These transformations act linearly on the two $(2n+2)$-component vectors

$$V = \begin{pmatrix} X^I \\ F_I(X,\hat{A}) \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} p^I \\ q_I \end{pmatrix}. \tag{2.2}$$

Here $F_I = \partial F/\partial X^I$, and p^I, q_I are the electric and magnetic charges of a given solution. We can use these vectors to define the central charge

$$Z = e^{\mathcal{K}/2}\left(p^I F_I(X,\hat{A}) - q_I X^I\right), \tag{2.3}$$

where $e^{-\mathcal{K}} = i[\bar{X}^I F_I(X,\hat{A}) - \bar{F}_I(\bar{X},\bar{\hat{A}})X^I]$, which is invariant under symplectic transformations.

When F depends on $\hat{A}$ the action contains higher curvature couplings between the Weyl and vector multiplets. The bosonic lagrangian contains the terms

$$8\pi\mathcal{L} = -\frac{1}{2}e^{-\mathcal{K}}R + \frac{1}{2}(iF_{\hat{A}}\,\hat{C} + \text{h.c.}) + \cdots, \tag{2.4}$$

where $\hat{C} = 64\,C^{-\mu\nu\rho\sigma}C^{-}_{\mu\nu\rho\sigma} + 16\,\varepsilon_{ij}\,T^{\mu\nu ij}\,f_\mu{}^\rho\,T_{\rho\nu kl}\,\varepsilon^{kl} + \cdots$. Here $C^{-}_{\mu\nu\rho\sigma}$ denotes the anti-selfdual part of the Weyl tensor $C_{\mu\nu\rho\sigma}$, and $f_\mu{}^\nu = \frac{1}{2}R_\mu{}^\nu - \frac{1}{12}R\,\delta_\mu{}^\nu + \cdots$. To fix the local scale invariance of the theory and obtain $N=2$ Poincaré supergravity, we must set $e^{-\mathcal{K}}$ to one. The first term in the action is then the properly normalized Einstein-Hilbert term.

2.2 Black hole solutions

Black hole solutions to this theory may be parameterized in terms of two functions $f(r)$ and $g(r)$ as

$$ds^2 = -e^{2g(r)}dt^2 + e^{2f(r)}\left[dr^2 + r^2(\sin^2\theta\,d\phi^2 + d\theta^2)\right]. \tag{2.5}$$

At $r\to\infty$, $e^{2g(r)}\to 1$ and $e^{2f(r)}\to 1$, so the metric is asymptotically flat. As $r\to 0$, it was shown by [4] that

$$e^{2f(r)} = e^{-2g(r)} = \frac{|Z\bar{Z}|_{r=0}}{r^2}, \tag{2.6}$$

where Z is the central charge defined above. When $Z|_{r=0}$ is non-vanishing the solution becomes $AdS_2\times S^2$ at small r, and the black hole has a finite horizon area

$$A = 4\pi|Z\bar{Z}|_{r=0}. \tag{2.7}$$

However, when $|Z\bar{Z}|_{r=0}=0$ the solution has a null singularity: typically $e^{2f(r)} = e^{-2g(r)} \sim 1/r$, and the horizon at $r=0$ is coincident with the null singularity.

Equation (2.7) may also be derived by imposing unbroken N=2 supersymmetry at the horizon. Starting from the gravitino variation

$$\delta\psi^i_\mu = 2\mathcal{D}_\mu\epsilon^i - \frac{1}{8}T^{ab\,ij}\gamma_{ab}\,\gamma_\mu\epsilon_j \tag{2.8}$$

the gauge invariant condition $\delta\mathcal{D}_{[\mu}\psi^i_{\nu]} = 0$ implies a relation between the curvature of a near horizon $AdS_2 \times S^2$ and the value of the central charge Z at $r = 0$. This procedure is explained in detail in [15].

To evaluate $Z|_{r=0}$, we may use the generalized attractor equations [4]

$$\bar{Z}\begin{pmatrix} X^I \\ F_I(X,\hat{A}) \end{pmatrix} - Z\begin{pmatrix} \bar{X}^I \\ \bar{F}_I(\bar{X},\bar{\hat{A}}) \end{pmatrix} = ie^{-\mathcal{K}/2}\begin{pmatrix} p^I \\ q_I \end{pmatrix} \tag{2.9}$$

which follow from the constraint of unbroken N=2 supersymmetry at the horizon. When there are no quantum corrections, i. e. when the prepotential F is independent of $\hat{A}$, these are the usual attractor equations [3]. When F depends on $\hat{A}$, consistency of the construction requires that $\hat{A} = -64\, e^{-\mathcal{K}}\, \bar{Z}^{-2}$.

The authors of [4] considered regular black hole solutions which classically have non-zero horizon area. In the presence of higher curvature corrections, the entropy is not given by the usual Bekenstein-Hawking formula (see, e.g. [16]). Instead, following Wald's proposal [7], the authors of [4] found

$$S = \pi\left[|Z|^2 - 256\,\mathrm{Im}\,[F_{\hat{A}}(X^I,\hat{A})]\right], \qquad \text{with} \qquad \hat{A} = -64\,\bar{Z}^{-2}\,\mathrm{e}^{-\mathcal{K}}, \tag{2.10}$$

where X^I and Z are fixed by the stabilization equations at $r = 0$. The first term is one quarter the horizon area (including higher derivative corrections), while the second term is the novel contribution to the entropy due to Wald.

Following [17], we will introduce the rescaled variables $Y^I = \mathrm{e}^{\mathcal{K}/2}\bar{Z}X^I$ and $\Upsilon = \mathrm{e}^{\mathcal{K}}\bar{Z}^2\hat{A}$. Using the homogeneity property of F, we find

$$Z\bar{Z} = p^I F_I(Y,\Upsilon) - q_I Y^I\,. \tag{2.11}$$

The stabilization equations (2.9) become

$$Y^I - \bar{Y}^I = ip^I\,, \tag{2.12}$$

$$F_I(Y,\Upsilon) - \bar{F}_I(\bar{Y},\bar{\Upsilon}) = iq_I\,. \tag{2.13}$$

The entropy formula (2.10) takes the form $S = \pi[Z\bar{Z} - 256\,\mathrm{Im}\,F_\Upsilon(Y,\Upsilon)]$ with $\Upsilon = -64$.

2.3 Calabi-Yau black holes

In this paper, we will focus on black holes found by compactifying type-IIA string theory on a large Calabi-Yau threefold. The prepotential has zero and linear order dependence on $\hat{A}$, so that

$$F(Y,\Upsilon) = \frac{D_{ABC}Y^AY^BY^C}{Y^0} + d_A\,\frac{Y^A}{Y^0}\,\Upsilon\,. \tag{2.14}$$

Here $I = 0,\ldots,n$, $A = 1,\ldots,n$ and

$$D_{ABC} = -\frac{1}{6}C_{ABC}\,, \qquad d_A = -\frac{1}{24}\,\frac{1}{64}\,c_{2A} \tag{2.15}$$

depend on the Calabi-Yau four-cycle intersection numbers C_{ABC} and second Chern-class coefficients c_{2A} [18]. The classical solution to the attractor equations, where $\Upsilon \to 0$ in (2.14), was found by Shmakova [19].

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The quantum corrected solution, including the second term in (2.14), is somewhat more complicated. In this case the lagrangian (2.4) contains a higher curvature term proportional to $c_{2A}\, z^A\, R^2$, where $z^A = Y^A/Y^0$. Let us take $p^0 = 0$ and assume that there is a solution with $Y^0 \neq 0$. The solution to the generalized attractor equation was found in this case to be [4]

$$Y^0 = \bar{Y}^0\,, \qquad (Y^0)^2 = \frac{D - 4 d_A\, p^A\, \Upsilon}{4\hat{q}_0}\,, \qquad Y^A = \frac{1}{6}\, Y^0\, D^{AB} q_B + \frac{1}{2} i\, p^A\,. \tag{2.16}$$

Here, following [19], we denote $D \equiv D_{ABC}\, p^A p^B p^C$, $D_{AB} \equiv D_{ABC}\, p^C$, and define D^{AB} by $D_{AB} D^{BC} \equiv \delta_A^C$. We have also introduced $\hat{q}_0 = q_0 + \frac{1}{12}\, D^{AB} q_A q_B$, which we will take to be negative. Note that this formula is valid only when the matrix $D_{AB} = D_{ABC} p^C$ is invertible. In the next section we will consider general configurations where this may not be the case. We must also take $p^A > 0$ to keep (Y^0) real.

Putting this together, we have the corrected formula for the horizon area

$$A = 4\pi |Z|^2_{r=0} = 16\pi Y^0 |\hat{q}_0| - 8\,\pi \frac{d_A\, p^A}{Y^0}\, \Upsilon\,. \tag{2.17}$$

The corrected entropy formula, using $\Upsilon = -64$, is

$$S = 4\pi\, Y^0 |\hat{q}_0|\,. \tag{2.18}$$

This agrees with the microscopic value computed in [8].

3. A stringy cloak for a null singularity

In this section we study supersymmetric solutions with vanishing classical horizon area, and conclude that quantum corrections due to R^2-terms lead to non-zero area.

Consider a Calabi-Yau black hole with vanishing graviphoton magnetic charge $p^0 = 0$. In the absence of higher curvature terms, i.e. taking $\Upsilon \to 0$, the area and entropy take the classical values

$$S_{cl} = \frac{A_{cl}}{4} = 4\pi\, Y^0_{cl} |\hat{q}_0| \; = 2\pi \sqrt{\hat{q}_0 D_{ABC} p^A p^B p^C}\,. \tag{3.1}$$

We have used here the attractor equations to fix the classical value of Y^0 at $r = 0$,

$$(Y^0)^2_{cl} = \frac{D}{4\hat{q}_0}\,. \tag{3.2}$$

We will consider black holes with charges p^A such that

$$D \equiv D_{ABC} p^A p^B p^C = 0 \tag{3.3}$$

vanishes. We see from (3.1) that the classical value for the area vanishes. Note that the classical value of Y^0 given by (3.2) vanishes, so the attractor solutions presented above become singular.

Fortunately, quantum corrections to the prepotential allow one to consistently solve the attractor equations. Making the ansatz

$$Y^A = w^A + \frac{i}{2} p^A \,, \tag{3.4}$$

where the w^A are real, the attractor equations become

$$D_{AB} w^B = \frac{1}{6} Y^0 q_A \,, \qquad D_{AB} w^A w^B = -\frac{1}{3} (q_0 (Y^0)^2 + d_{A} p^A \Upsilon) \,. \tag{3.5}$$

When D_{AB} is invertible, the solution is given by (2.16),

$$Y^0 = \sqrt{\frac{c_{2A}\, p^A}{24 |\hat{q}_0|}} \,. \tag{3.6}$$

The quantum corrected horizon area becomes

$$A = 4\pi |Z|^2_{r=0} = 8\pi \sqrt{\frac{c_{2A}\, p^A |\hat{q}_0|}{24}} \,. \tag{3.7}$$

This is non-zero provided $c_{2A} p^A$ and $\hat{q}_0$ are non-vanishing. Note that A depends on the topology of the Calabi-Yau (via the Chern classes c_{2A}) as well as on the charges of the black hole. The area is large when $\hat{q}_0$, $c_{2A} p^A$ are large. Formula (3.7) may change once higher order α' and g_s corrections are included.

If the matrix D_{AB} is not invertible, then the solution is a little more complicated. In this case, as may be seen from (3.5), one must impose additional conditions on the electric charges q_A to keep Y^0 non-zero. The simplest constraint is $q_A = 0$, in which case the above solution is correct provided we set $\hat{q}_0 = q_0$. More generally, the charge vector q_A must be orthogonal to the kernel of the matrix D_{AB}. In this case we have $\hat{q}_0 = q_0 + \frac{1}{12} \hat{D}^{AB} q_A q_B$, where $\hat{D}^{AB}$ is the inverse of D_{AB} in the subspace orthogonal to it's kernel.

For all of the configurations described above, the corrected entropy is

$$S = 4\pi\, Y^0 |\hat{q}_0| = 4\pi \sqrt{\frac{c_{2A}\, p^A |\hat{q}_0|}{24}} \,. \tag{3.8}$$

The quantum corrected entropy-area relation for this class of black holes is

$$S = \frac{A}{2} \,. \tag{3.9}$$

We should emphasize that we have derived this relation only for this specific class of solutions. Both S and A are symplectic invariants, so it is not surprising that they remain proportional even after higher order corrections are included, but this specific relation is not generic. It would be interesting to find the corresponding relation for other types of black holes.

4. Two-charge Black Holes in $K_3 \times T^2$ Compactification

In this section we specialize to Type-IIA string theory compactified on $K_3 \times T^2$ considered in [10]. There are 23 two-cycles, provided by the torus T^2 and the 22 two-cycles of K_3. The non-vanishing intersection numbers are

$$C_{1ab} = C_{ab}\,, \tag{4.1}$$

where $a = 2, \ldots, 23$ runs over the two-cycles of K_3 and C_{ab} is the intersection matrix on K_3. The second Chern class is $c_{2,1} = 24$.

One virtue of this special example is that the power series expansion of the holomorphic function $F(X^I, \hat{A})$ terminates at first order in $\hat{A}$. Moreover, the full quantum corrected expression for F can be evaluated exactly (see e.g. [10]). This exact expression for F includes the terms listed in equation (2.14), as well as corrections that are suppressed when the volume of $K_3 \times T^2$ is large. This allows a more detailed comparison of the microscopic degeneracy with the black hole partition function defined by [9] to all orders in the inverse charge in an asymptotic expansion.

We consider the configuration where $q_0 = n$ and $p^1 = w$ are non-zero and the rest of the charges vanish. This corresponds to a configuration of w D4-branes wrapping the K_3 with n D0-branes sprinkled on the worldvolume. This state is dual to a perturbative state with winding number w and quantized momentum n in the heterotic theory compactified on $T^4 \times T^2$. The condition (3.3) is satisfied for this configuration, so the black holes classically have zero area. The quantum corrected area is

$$A = 8\pi\sqrt{p^1|q_0|} \tag{4.2}$$

and the corrected entropy is

$$S = 4\pi\sqrt{p^1|q_0|} = \frac{A}{2}\,. \tag{4.3}$$

The corrected geometry of the black hole near the horizon is given by $AdS_2 \times S^2$. In Einstein frame, the radius of the sphere goes as $N^{\frac{1}{4}}$, where $N \equiv |nw|$ can be taken to be large. The attractor equations also determine the value of the heterotic dilaton ϕ at the horizon. Recall that $e^{-2\phi}$ is given by the imaginary part of Y^1/Y^0, which gives $e^{-2\phi} \sim \sqrt{N}$ from the solution of the attractor equations for our charge configuration [10]. The closed-string loop-counting parameter $g_s^2 = e^{2\phi}$ thus varies in the black hole geometry and reaches a constant value at the horizon that goes as $\frac{1}{\sqrt{N}}$, and is thus vanishingly small for large N.

Given this value of the dilaton, the radius of curvature at the horizon is of order string scale in the string frame even though it is large in Einstein frame. This is consistent with the heuristic idea of a string-scale 'stretched horizon' [20] for the zero area extremal black holes [21] in the heterotic description. Note that while the string metric determines the sigma-model couplings, the Einstein metric is the one that is invariant under duality.

In the Type-IIB description, this system is dual to the D1-D5 system where the geometry is known to be desingularized already at the classical level without the inclusion of higher curvature terms [22, 23]. It was found that there are many such smooth geometries,

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which are dual to the oscillating string solutions of [24, 25] that are specified by an arbitrary profile of left-moving oscillations. The multiplicity of these smooth solutions then accounts for the degeneracy of the corresponding BPS states. We have found instead that the inclusion of higher curvature terms generates a regular horizon, and the entropy associated with this horizon accounts for the degeneracy of these states. It would be interesting to understand in detail how these two dual pictures are compatible with each other.

The entropy of these black holes no longer satisfies the Bekenstein-Hawking area formula but also includes the correction due to higher derivative terms given by Wald's formula. For the class of solutions considered here, we find that the entropy is given by $S = A/2 \equiv A/4 + A/4$. Thus the correction to the entropy is of the same order as the Bekenstein-Hawking entropy. Note that A here is the quantum corrected area and is quite different from the classical area which vanishes.

This raises an important physical question. The Bekenstein-Hawking formula for the entropy allows one to view the area theorems of black hole dynamics in classical general relativity as special cases of black hole thermodynamics. In particular, the fact that the area of black holes always increases in physical processes can then be viewed as a special case of the second law of thermodynamics that the entropy always increases in physical processes. For our black holes, since the entropy is quite different from classical area, it is important to show that the geometric quantity defined as the entropy by Wald's formula always increases in a physical process. This is necessary to ensure that the second law of thermodynamics is obeyed with the definition of entropy by Wald which appears in the first law of black hole thermodynamics. Note that the entropy thus defined is in perfect agreement with the counting of BPS microstates and the specific corrections to the Einstein-Hilbert term used here are well-motivated by string theory. Therefore, slightly non-extremal black holes in this system could be a good starting point for considering possible generalizations of area theorems of classical general relativity within string theory.

5. Conclusions

In conclusion, we have found a large family of classically singular, zero area solutions which are converted into regular black holes by higher derivative corrections to the supergravity action. This class includes the two-charge solutions of string theory on $K_3 \times T^2$ studied recently in [10].

The entropy-area relation for this particular class of black holes is $S = A/2$, which differs from the well known classical relation $S = A/4$. However, we expect that this modified version the Bekenstein-Hawking formula applies only to this special class of Calabi-Yau black holes.

It would be interesting to investigate other black hole solutions with vanishing classical horizon area. We would also like to understand any relation between our stringy cloaking device and the proposal of [9], as well as the effects of higher order α' and g_s corrections on our results. We leave this for future work.

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References

[1] B. de Wit, J.W. van Holten and A. Van Proeyen, *Transformation rules of $N = 2$ supergravity multiplets*, *Nucl. Phys.* **B 167** (1980) 186; *Structure of $N = 2$ supergravity*, *Nucl. Phys.* **B 184** (1981) 77;
B. de Wit and A. Van Proeyen, *Potentials and symmetries of general gauged $N = 2$ supergravity - Yang-Mills models*, *Nucl. Phys.* **B 245** (1984) 89;
B. de Wit, P.G. Lauwers and A. Van Proeyen, *Lagrangians of $N = 2$ supergravity-matter systems*, *Nucl. Phys.* **B 255** (1985) 569.

[2] S. Ferrara, R. Kallosh and A. Strominger, *N=2 extremal black holes*, *Phys. Rev.* **D 52** (1995) 5412 [hep-th/9508072];
A. Strominger, *Macroscopic entropy of $n = 2$ extremal black holes*, *Phys. Lett.* **B 383** (1996) 39 [hep-th/9602111].

[3] S. Ferrara and R. Kallosh, *Supersymmetry and attractors*, *Phys. Rev.* **D 54** (1996) 1514 [hep-th/9602136]; *Universality of supersymmetric attractors*, *Phys. Rev.* **D 54** (1996) 1525 [hep-th/9603090].

[4] G. Lopes Cardoso, B. de Wit and T. Mohaupt, *Corrections to macroscopic supersymmetric black-hole entropy*, *Phys. Lett.* **B 451** (1999) 309 [hep-th/9812082]; *Deviations from the area law for supersymmetric black holes*, *Fortschr. Phys.* **48** (2000) 49 [hep-th/9904005]; *Macroscopic entropy formulae and non-holomorphic corrections for supersymmetric black holes*, *Nucl. Phys.* **B 567** (2000) 87 [hep-th/9906094].

[5] G.L. Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, *Examples of stationary BPS solutions in $N = 2$ supergravity theories with $\mathbb{R}^2$-interactions*, *Fortschr. Phys.* **49** (2001) 557 [hep-th/0012232]; *Supersymmetric black hole solutions with $\mathbb{R}^2$ interactions*, hep-th/0003157;
G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, *Stationary BPS solutions in $N = 2$ supergravity with $\mathbb{R}^2$ interactions*, *J. High Energy Phys.* **12** (2000) 019 [hep-th/0009234].

[6] S.W. Hawking, *Gravitational radiation from colliding black holes*, *Phys. Rev. Lett.* **26** (1971) 1344; *Particle creation by black holes*, *Commun. Math. Phys.* **43** (1975) 199;
J.M. Bardeen, B. Carter and S.W. Hawking, *The four laws of black hole mechanics*, *Commun. Math. Phys.* **31** (1973) 161;
J.D. Bekenstein, *Black holes and entropy*, *Phys. Rev.* **D 7** (1973) 2333; *Generalized second law of thermodynamics in black hole physics*, *Phys. Rev.* **D 9** (1974) 3292.

[7] R.M. Wald, *Black hole entropy in the noether charge*, *Phys. Rev.* **D 48** (1993) 3427 [gr-qc/9307038];
T. Jacobson, G. Kang and R.C. Myers, *On black hole entropy*, *Phys. Rev.* **D 49** (1994) 6587 [gr-qc/9312023]; *Black hole entropy in higher curvature gravity*, gr-qc/9502009;

V. Iyer and R.M. Wald, *Some properties of noether charge and a proposal for dynamical black hole entropy*, *Phys. Rev.* **D 50** (1994) 846 [gr-qc/9403028];
R.M. Wald, *Black holes and thermodynamics*, gr-qc/9702022; *Gravitation, thermodynamics and quantum theory*, *Class. and Quant. Grav.* **16** (1999) A177 [gr-qc/9901033].

[8] J.M. Maldacena, A. Strominger and E. Witten, *Black hole entropy in M-theory*, *J. High Energy Phys.* **12** (1997) 002 [hep-th/9711053];
C. Vafa, *Black holes and Calabi-Yau threefolds*, *Adv. Theor. Math. Phys.* **2** (1998) 207 [hep-th/9711067].

[9] H. Ooguri, A. Strominger and C. Vafa, *Black hole attractors and the topological string*, *Phys. Rev.* **D 70** (2004) 106007 [hep-th/0405146].

[10] A. Dabholkar, *Exact counting of black hole microstates*, hep-th/0409148.

[11] A. Dabholkar and J.A. Harvey, *Nonrenormalization of the superstring tension*, *Phys. Rev. Lett.* **63** (1989) 478.

[12] A. Dabholkar, G.W. Gibbons, J.A. Harvey and F. Ruiz Ruiz, *Superstrings and solitons*, *Nucl. Phys.* **B 340** (1990) 33.

[13] C.V. Johnson, A.W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, *Phys. Rev.* **D 61** (2000) 086001 [hep-th/9911161].

[14] T. Mohaupt, *Black hole entropy, special geometry and strings*, *Fortschr. Phys.* **49** (2001) 3 [hep-th/0007195].

[15] A.H. Chamseddine, S. Ferrara, G.W. Gibbons and R. Kallosh, *Enhancement of supersymmetry near 5d black hole horizon*, *Phys. Rev.* **D 55** (1997) 3647 [hep-th/9610155].

[16] T. Jacobson, G. Kang and R.C. Myers, *Increase of black hole entropy in higher curvature gravity*, *Phys. Rev.* **D 52** (1995) 3518 [gr-qc/9503020];
R.C. Myers, *Black holes in higher curvature gravity*, gr-qc/9811042.

[17] K. Behrndt et al., *Classical and quantum $N = 2$ supersymmetric black holes*, *Nucl. Phys.* **B 488** (1997) 236 [hep-th/9610105].

[18] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, *Holomorphic anomalies in topological field theories*, *Nucl. Phys.* **B 405** (1993) 279 [hep-th/9302103].

[19] M. Shmakova, *Calabi-Yau black holes*, *Phys. Rev.* **D 56** (1997) 540 [hep-th/9612076].

[20] L. Susskind, *Strings, black holes and Lorentz contraction*, *Phys. Rev.* **D 49** (1994) 6606 [hep-th/9308139].

[21] A. Sen, *Black hole solutions in heterotic string theory on a torus*, *Nucl. Phys.* **B 440** (1995) 421 [hep-th/9411187]; *Extremal black holes and elementary string states*, *Mod. Phys. Lett.* **A 10** (1995) 2081 [hep-th/9504147].

[22] O. Lunin and S.D. Mathur, *AdS/CFT duality and the black hole information paradox*, *Nucl. Phys.* **B 623** (2002) 342 [hep-th/0109154].

[23] O. Lunin, J. Maldacena and L. Maoz, *Gravity solutions for the D1-D5 system with angular momentum*, hep-th/0212210.

[24] C.G. Callan Jr., J.M. Maldacena and A.W. Peet, *Extremal black holes as fundamental strings*, *Nucl. Phys.* **B 475** (1996) 645 [hep-th/9510134].

[25] A. Dabholkar, J.P. Gauntlett, J.A. Harvey and D. Waldram, *Strings as solitons & black holes as strings*, *Nucl. Phys.* **B 474** (1996) 85 [hep-th/9511053].